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ON QUANTUM NATURE OF BLACK-HOLE SPACETIME: A Possible New Source of Intense Radiation *

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Atoms and the planets acquire their stability from the quantum mechanical incompatibility of the position and momentum measurements. This incompatibility is expressed by the fundamental commutator $[x, p_x] = i\hbar$, or equivalently, via the Heisenberg's uncertainty principle $\Delta x \Delta p_x \sim \hbar$. A further stability-related phenomenon where the quantum realm plays a dramatic role is the collapse of certain stars into white dwarfs and neutron stars. Here, an intervention of the Pauli exclusion principle, via the fermionic degenerate pressure, stops the gravitational collapse. However, by the neutron-star stage the standard quantum realm runs dry. One is left with the problematic collapse of a black hole. This essay is devoted to a concrete argument on why the black-hole spacetime itself should exhibit a quantum nature. The proposed quantum aspect of spacetime is shown to prevent the general-relativistic dictated problematic collapse. The quantum nature of black-hole spacetime is deciphered from a recent result on the universal equal-area spacing $[= \lambda_P^2 4 \ln(3)]$ for black holes. In one interpretation of the emergent picture, an astrophysical black hole can fluctuate to $\sqrt{\pi/\ln(3)}$ (≈ 1.7) time its classical size, and thus allow radiation and matter to escape to the outside observers. These fluctuations I conjecture provide a new source, perhaps beyond Hawking radiation, of intense radiation from astrophysical black holes and may be the primary source of observed radiation from those galactic cores what carry black hole(s). The presented interpretation may be used as a criterion to choose black holes from black hole candidates.

1. For more than two decades theoretical arguments have been accumulating in favor of black holes having a discrete surface area. Insightful reviews on the subject include those by Ashtekar¹ and Bekenstein.² In Ref. [2], in support of the uniform spacing for the discrete area eigenstates, Bekenstein notes that, *“transition frequencies at large quantum numbers should equal classical oscillation frequencies”, because a classical Schwarzschild black hole displays ‘ringing frequencies’ which scale as M^{-1} ... This agreement would be destroyed if the area eigenvalues were unevenly spaced. Indeed, the loop gravity spectrum ... fails this correspondence principle test.* Earlier works that obtained uniformly spaced area eigenstates for excitations of black holes include string theoretic argument of Kogan,³ and the quantum membrane approaches of Maggiore⁴ and Lousto⁵, and canonical quantum gravity approaches of Louko and Mäkelä,⁶ among others. However, there is no general agreement on the spacing of the area eigenstates, or on its uniformity.

This situation has been changed to an extent. In a recent work Hod has argued

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that the black-hole uniform area spacing is given by⁷

$$\Delta\mathcal{A}' = \lambda_P^2 4 \ln(3) \quad \left[\lambda_P \equiv (\hbar G/c^3)^{1/2}, \text{ the Planck length} \right], \quad (1)$$

Furthermore, Hod has shown that the area spacing $\lambda_P^2 4 \ln(3)$ is the *unique* value consistent with both the area-entropy thermodynamic relation, with the statistical physics arguments (i.e., with the Boltzmann-Einstein relation), and with the Bohr's correspondence principle. The *universality* of this result suggests that there is something fundamental that underlies the equal area spacing. This essay is an *ab initio* attempt towards discerning this hint. I will explore the consequences of the conjecture that the uniform area spacing is an intrinsic property of the black hole spacetime and holds even before the Bohr correspondence limit is reached.

2. The simplest black-hole area operator, compatible with the stated conjecture, resides in the observation that the equal energy-level spacing of a one-dimensional quantum harmonic oscillator (*QHO*) arise as a consequence of the fundamental commutator $[x, p_x] = i\hbar$ and from the fact that *QHO* Hamiltonian operator consists, additively, the operators x^2 and p_x^2 . Thus the simplest area operator compatible with the stated conjecture is

$$\mathcal{A} = 2\pi \ell_S^2 + \frac{2 [\ln(3)]^2}{\pi} \ell_P^2, \quad (2)$$

The ‘‘Schwarzschild’’ (S) and the ‘‘Planck’’ (P) operators satisfy the following fundamental commutator

$$[\ell_P, \ell_S] = i\lambda_P^2. \quad (3)$$

The factor of 2π in the first term on the right hand side of Eq. (2) owes its origin to a remark after Eq. (9) below.

The stated conjecture thus carries an element of a unifying thread in that it extends the quantum-harmonic-oscillator like structure that has been so successful in understanding the non-gravitational fields to the black-hole spacetime itself. It is possible that the two terms on the right hand side of Eq. (2) are connected via a worm hole (whose contribution to the area operator is presumably negligible and is missing from Eq. (2), but which provides a traversable throat). The other possibility is that quantum fluctuations of black hole spacetime make the ‘‘compactified dimensions’’ physically accessible and result in the indicated bifurcation of the area operator. In either case, Eq. (3) represents fundamental restriction on the simultaneous observability of the relevant sectors — symbolically represented by ‘‘S’’ and ‘‘P.’’ These speculations on the physical interpretation have, in part, been inspired by the wormhole papers of Kim and Lee,⁸ and Garattini,⁹ on the one hand, and a large literature that exists on the non-commutative aspects of spacetime at the Planck scale on the other (see Ref. [10] for a brief historical review). The latter aspect is an unavoidable consequence of the fact that gravitational effects, associated with the quantum measurements of spacetime intervals, render the physical spacetime noncommutative.¹¹

It is of interest to note that Amelino-Camelia^{14,15} has recently argued that an interferometric gravitational wave detector can be used as a quantum-gravity apparatus to probe the fuzzy/foamy picture of spacetime. The latter picture of spacetime is a prediction of nearly all approaches that hope to combine gravitation and quantum mechanics. Thus, an experimentally accessible quantum-gravity spacetime emerges in these pictures where one aspect is governed by a source-determined length scale, $\lambda_S \simeq 2GM/c^2$; and the other one is governed by the source-independent length scale

$\lambda_P = (\hbar G/c^3)^{1/2}$. The former aspect is associated with the spacetime structure in general relativity, while the latter one is a characteristic of some (yet unknown, and the realm of the stated conjecture) fundamental nature of spacetime. The conjecture put forward above incorporates both elements in a natural way. As already noted, the *compactified dimensions* that are central to almost all current theories (strings, membranes, and so on) may live as quantum mechanically incompatible dimensions to our four dimensional spacetime and are phenomenologically described by postulates (2) and (3).

Parenthetically, a note is in place regarding a recent paper by Padmanabhan who showed that a modification of the path integral based on the principle of duality (i.e., invariance of the path integral amplitude under ds , the path segment, going from $ds \rightarrow \lambda_P^2/ds$) leads to results which are identical to adding a “zero-point length” in the spacetime interval.¹⁶ The equivalence of the duality invariance, and that of the transformation of $g_{\mu\nu}dx^\mu dx^\nu \rightarrow g_{\mu\nu}dx^\mu dx^\nu + \lambda_P^2$, is akin to the postulated premise here.

The presented considerations are confined to a Schwarzschild black hole.

3. It is now natural to introduce the quantum operators d and d^\dagger as

$$d = \frac{1}{\lambda_P} \sqrt{\frac{\pi}{2 \ln(3)}} \left(\ell_S - i \frac{\ln(3)}{\pi} \ell_P \right), \quad d^\dagger = \frac{1}{\lambda_P} \sqrt{\frac{\pi}{2 \ln(3)}} \left(\ell_S + i \frac{\ln(3)}{\pi} \ell_P \right). \quad (4)$$

From the fundamental commutator $[\ell_P, \ell_S] = i\lambda_P^2$ it follows that $[d, d^\dagger] = \mathbf{1}$. Inverting the above equations for ℓ_S and ℓ_P , and introducing the area number operator $\eta = d^\dagger d$, the area operator (2) takes the form

$$\mathcal{A} = \lambda_P^2 4 \ln(3) \left(\eta + \frac{1}{2} \right). \quad (5)$$

The eigenvalues of η , $\eta|\eta'\rangle = \eta'|\eta'\rangle$, are zero and positive integers. As a consequence, the black hole area spectrum is quantized

$$\mathcal{A}'_{\eta'} = \lambda_P^2 4 \ln(3) \left(\eta' + \frac{1}{2} \right) \quad [\eta' = 0, 1, 2, \dots], \quad (6)$$

and carries with it a fundamental *zero-point area*

$$\mathcal{A}'_0 = \frac{1}{2} \lambda_P^2 4 \ln(3). \quad (7)$$

Since

$$d|\eta'\rangle = \sqrt{\eta'} |\eta' - 1\rangle, \quad d^\dagger|\eta'\rangle = \sqrt{\eta' + 1} |\eta' + 1\rangle, \quad (8)$$

the d and d^\dagger obtain the interpretation of the *area annihilation* and *area creation* operators.

The postulated fundamental commutator, and the area operator, find their initial justification in preventing a black hole from collapsing into the general-relativistic dictated singularity, while at the same time reproducing the equal-area spacing.⁷

4. It is readily verified that the following relations are valid:

$$\langle \eta' | \ell_P^2 | \eta' \rangle = \lambda_P^2 \frac{\pi}{\ln(3)} \left(\eta' + \frac{1}{2} \right), \quad \langle \eta' | \ell_S^2 | \eta' \rangle = \lambda_P^2 \frac{\ln(3)}{\pi} \left(\eta' + \frac{1}{2} \right). \quad (9)$$

As a result, the area expectation value $\langle \eta' | \mathcal{A} | \eta' \rangle = \mathcal{A}'_{\eta'}$ derives equal contribution from the Schwarzschild and the Planck sectors of the area operator. Each of these contributions to $\mathcal{A}'_{\eta'}$ equals

$$\lambda_P^2 2 \ln(3) \left(\eta' + \frac{1}{2} \right). \quad (10)$$

Now consider $\eta' \gg 0$, and call such black holes astrophysical black holes. In the spirit of the Bohr's correspondence principle, I equate $\lambda_P^2 4 \ln(3) \eta'_{astro}$ to the classical result, $4\pi\lambda_S^2$. This yields

$$\eta'_{astro} = \frac{\pi}{\ln(3)} \left(\frac{\lambda_S}{\lambda_P} \right)^2. \quad (11)$$

With the help of relations (9), I arrive at the quantum-gravity uncertainty relation inherent in the postulated premise

$$(\Delta \ell_P)_{\eta'} (\Delta \ell_S)_{\eta'} \geq \left(\eta' + \frac{1}{2} \right) \lambda_P^2 \quad [\eta' = 0, 1, 2, \dots], \quad (12)$$

The quantum fluctuations that appears in the above-derived quantum-gravity uncertainty relation, are defined as:

$$(\Delta \ell_\zeta)_{\eta'}^2 \equiv \langle \eta' | \ell_\zeta^2 | \eta' \rangle - \langle \eta' | \ell_\zeta | \eta' \rangle^2 \quad [\zeta = S, P]. \quad (13)$$

Thus, in the Bohr's correspondence regime, on using Eq. (11), it follows that

$$(\Delta \ell_P)_{\eta'} (\Delta \ell_S)_{\eta'} \geq (\pi / \ln(3)) \lambda_S^2 \quad [\eta' = \eta'_{astro}]. \quad (14)$$

The $\sqrt{(\Delta \ell_P)_{\eta'} (\Delta \ell_S)_{\eta'}}$ may be interpreted to provide a rough measure of the quantum size-fluctuations for a black hole. With this interpretation, I infer that quantum fluctuations can allow astrophysical black holes to fluctuate to a size $\sqrt{\pi / \ln(3)} \approx 1.7$ times their classical size. This allows radiation and matter to escape to the outside observers. I know of no reason to discount this as the primary source of intense radiation from those galactic cores that carry black hole(s).

The picture of the quantum-gravity spacetime associated with an astrophysical black hole that emerges is that of an object with two quantum spheres of fluctuations. The one that may be called a *Schwarzschild sphere*, and the other a *Planck sphere*. The sizes of these two spheres may be characterized by [and are obtained on combining Eqs. (9) and (11)]

$$\eta' = \eta'_{astro} : \quad (\Delta \ell_S)_{\eta'} = \lambda_S, \quad (15)$$

$$(\Delta \ell_P)_{\eta'} = (\pi / \ln(3)) \lambda_S \approx 2.86 \lambda_S, \quad (16)$$

As a cautionary remark I note that the vanishing of $\langle \eta' | \ell_S | \eta' \rangle$ and $\langle \eta' | \ell_P | \eta' \rangle$ does not imply that the associated quantum mechanical length scale is zero. All it

means is that ℓ_S and ℓ_P should be treated as (vector) “momenta” and “amplitudes.” Good measures of the quantum length scales of a black hole are $\sqrt{\langle \eta' | \ell_S^2 | \eta' \rangle}$ and $\sqrt{\langle \eta' | \ell_P^2 | \eta' \rangle}$. The latter, because $\langle \eta' | \ell_S | \eta' \rangle = 0 = \langle \eta' | \ell_P | \eta' \rangle$, become identical to $(\Delta \ell_S)_{\eta'}$ and to $(\Delta \ell_P)_{\eta'}$.

That the quantum-gravity space time must have some sort of fuzzy/foamy structure is not new. What is new, and is the subject of the present essay, is a precise model of this fuzzy/foamy structure with a predictive and calculational power. Emergence of the zero-point area is only one of the physical implications. Interestingly, the quantum-gravity fluctuations for the low-mass black holes (i.e., for η' close to zero) are determined by the Planck length λ_P as is apparent from an inspection of Eq. (12). On the other hand, as follows from Eq. (14), the quantum-gravity fluctuations for astrophysical black holes are governed by the Schwarzschild length, λ_S .

For Planck mass black holes if one identifies ℓ_S with the operator x , then the uncertainty Δx , apart from being restricted by the relation $\Delta x \Delta p_x \geq \hbar/2$, is further constrained to satisfy (according to Eq. 12)

$$\Delta x \geq \left(\eta' + \frac{1}{2} \right) \frac{\lambda_P^2}{(\Delta \ell_P)_{\eta'}}. \quad (17)$$

Thus, suggesting that the quantum-gravity effects shall require modification of the fundamental uncertainty relations of the standard quantum mechanics. Such modifications have already been suspected and argued for by several authors [11, 17-24] and carry important consequences for the theories that incorporate gravity and quantum mechanics.

6. I thus conclude that an important physical consequence of the conjectured universal equal-area spacing $\lambda_P^2 4 \ln(3)$ is that the quantum gravity description of the Schwarzschild black hole spacetimes is characterized by a quantum area operator (Eq. 2) and a new fundamental commutator (Eq. 3). The resulting emergence of the zero point area forbids a black hole from its collapse into the general-relativistic dictated singularity. An interpretation of the Schwarzschild and Planck spheres of quantum fluctuations is that they give black holes a size that is roughly $\sqrt{\pi/\ln(3)}$ (≈ 1.7) times their classical size. This allows radiation and matter to escape from a black hole to an outside observer perhaps dramatically beyond the Hawking radiation. Apart from astrophysical consequences, the possible existence of two mutually incompatible spatial dimensions has important cosmological consequences and it has serious impact on arguments on information loss in the context of black holes. Thus, the interpretation of the universal equal-area spacing $\lambda_P^2 4 \ln(3)$ as implying a quantum-harmonic-oscillator like structure of spacetime is the simplest theoretical construct that unifies gravitation to other non-gravitational interactions in a non trivial manner. The proposed interpretation carries in it deep seeds for new studies of the various paradoxes and problems associated with general-relativistic black holes and cosmology. The conjecture that *compactified dimensions* which are so central to almost all current theories embedded in higher dimensional spacetimes may live as quantum mechanically incompatible dimensions to our four dimensional spacetime provides an additional possibility to discern a new physical principle that may constrain the compactification procedure dramatically and may even redefine compactification itself.

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Note Added

Reader's attention is directed to Ref. [25] which appeared on the *LANL archives* after the present essay was accepted for publication. Several new references on the gravitationally induced modification to the uncertainty relations also appeared in print while this manuscript was under review. The references have been updated accordingly.

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